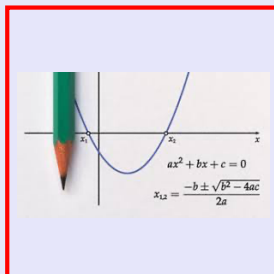


Math 125
Spring 2022
Lecture 20



Simplify

$$1) \sqrt{6}(6 + \sqrt{6}) = 6\sqrt{6} + \sqrt{36} = \boxed{6\sqrt{6} + 6}$$

$$2) (5 + \sqrt{2})(6 - \sqrt{2}) = 30 - 5\sqrt{2} + 6\sqrt{2} - \sqrt{4}$$

$$= 30 + 1\sqrt{2} - 2 = \boxed{28 + \sqrt{2}}$$

$$3) (\sqrt{10} - \sqrt{7})(\sqrt{10} + \sqrt{7}) =$$

$$= \sqrt{100} + \cancel{\sqrt{70}} - \cancel{\sqrt{70}} - \sqrt{49} = 10 - 7 = \boxed{3}$$

$$4) (2\sqrt{5} - \sqrt{3})^2 = (2\sqrt{5} - \sqrt{3})(2\sqrt{5} - \sqrt{3})$$

$$= 4\sqrt{25} - 2\sqrt{15} - 2\sqrt{15} + \sqrt{9}$$

$$= 4 \cdot 5 - 4\sqrt{15} + 3 = \boxed{23 - 4\sqrt{15}}$$

Solve & check:

1) $\sqrt{3x+2}=4$
 $(\sqrt{3x+2})^2=(4)^2$
 $3x+2=16$
 $3x=14$
 $x=\frac{14}{3}$ $x=\frac{14}{3}$

check
 $\sqrt{3(\frac{14}{3})+2}=4$
 $\sqrt{14+2}=4$
 $\sqrt{16}=4\checkmark$ $\{\frac{14}{3}\}$

2) $\sqrt{3x-2}-5=0$
 $\sqrt{3x-2}=5$
 $(\sqrt{3x-2})^2=(5)^2$
 $3x-2=25$

$\rightarrow 3x=27$
 $x=9$ $\{9\}$

check:
 $\sqrt{3(9)-2}-5=0$
 $\sqrt{27-2}-5=0$
 $\sqrt{25}-5=0$
 $5-5=0\checkmark$

3) $\sqrt{2x-5}-\sqrt{x+4}=0$
 $\sqrt{2x-5}=\sqrt{x+4}$
 $(\sqrt{2x-5})^2=(\sqrt{x+4})^2$

$\rightarrow 2x-5=x+4$ $\{9\}$
 $x=9$ check:
 $\sqrt{2(9)-5}-\sqrt{9+4}=0$
 $\sqrt{13}-\sqrt{13}=0\checkmark$

4) $\sqrt[3]{2x-6}-4=0$ check:
 $\sqrt[3]{2x-6}=4$
 $(\sqrt[3]{2x-6})^3=(4)^3$
 $2x-6=64$
 $2x=70$
 $x=35$ $x=35$

$\sqrt[3]{2(35)-6}-4=0$
 $\sqrt[3]{70-6}-4=0$
 $\sqrt[3]{64}-4=0$
 $4-4=0\checkmark$ $\{35\}$

4) Solve & check $\sqrt{2x+1} + x = 7$

$\sqrt{2x+1} = 7-x$
 $(\sqrt{2x+1})^2 = (7-x)^2$
 $2x+1 = (7-x)(7-x)$

$\rightarrow 2x+1 = 49 - 7x - 7x + x^2$
 $2x+1 = 49 - 14x + x^2$
 $0 = 49 - 14x + x^2 - 2x - 1$
 $0 = x^2 - 16x + 48$

check $x=4$
 $\sqrt{2(4)+1} + 4 = 7$
 $\sqrt{9} + 4 = 7\checkmark$

check $x=12 \rightarrow$ Extraneous Solution
 $\sqrt{2(12)+1} + 12 = 7$
 $\sqrt{25} + 12 = 7$
 $5 + 12 = 7$
 $17 = 7$
 False

$0 = (x-4)(x-12)$
 $x-4=0$ $x-12=0$
 $x=4$ $x=12$

$\{4\}$

Solve & check:

$$x - \sqrt{x-1} = 1$$

$$x - 1 = \sqrt{x-1}$$

$$(x-1)^2 = (\sqrt{x-1})^2$$

$$(x-1)(x-1) = x-1$$

$$x^2 - x - x + 1 = x-1$$

$$x^2 - 2x + 1 - x + 1 = 0$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$\downarrow \quad \downarrow$$

$$\checkmark \boxed{x=1} \quad \boxed{x=2} \checkmark$$

$$\{1, 2\}$$

Solve & check:

$$\sqrt{x-7} + \sqrt{x} = 7$$

$$\sqrt{x-7} = 7 - \sqrt{x}$$

$$(\sqrt{x-7})^2 = (7 - \sqrt{x})^2$$

$$x-7 = (7 - \sqrt{x})(7 - \sqrt{x})$$

$$x-7 = 49 - 7\sqrt{x} - 7\sqrt{x} + \sqrt{x}^2$$

$$x-7 = 49 - 14\sqrt{x} + x$$

$$-7 = 49 - 14\sqrt{x}$$

$$14\sqrt{x} = 49 + 7$$

$$14\sqrt{x} = 56$$

$$\sqrt{x} = \frac{56}{14}$$

$$\sqrt{x} = 4$$

$$(\sqrt{x})^2 = (4)^2$$

$$\boxed{x=16}$$

Check

$$\sqrt{16-7} + \sqrt{16} = 7$$

$$\sqrt{9} + \sqrt{16} = 7$$

$$3 + 4 = 7 \checkmark$$

$$\{16\}$$

Solve & check:

$$\sqrt{x-8} - \sqrt{x} = -2$$

$$\sqrt{x-8} = \sqrt{x} - 2$$

$$(\sqrt{x-8})^2 = (\sqrt{x} - 2)^2$$

$$x-8 = (\sqrt{x} - 2)(\sqrt{x} - 2)$$

check

$$\sqrt{9-8} - \sqrt{9} = -2$$

$$1 - 3 = -2 \checkmark$$

$$x-8 = \sqrt{x^2 - 2\sqrt{x} - 2\sqrt{x} + 4}$$

$$\cancel{x}-8 = \cancel{x} - 4\sqrt{x} + 4$$

$$-8 = -4\sqrt{x} + 4$$

$$4\sqrt{x} = 4 + 8$$

$$4\sqrt{x} = 12$$

$$\sqrt{x} = 3$$

$$\{9\}$$

$$\boxed{x=9} \checkmark$$

Solve

$$\sqrt{2x-3} - \sqrt{x-2} = 1$$

$$\sqrt{2x-3} = \sqrt{x-2} + 1$$

$$(\sqrt{2x-3})^2 = (\sqrt{x-2} + 1)^2$$

$$2x-3 = (\sqrt{x-2} + 1)(\sqrt{x-2} + 1)$$

$$2x-3 = (\sqrt{x-2})^2 + \sqrt{x-2} + \sqrt{x-2} + 1$$

$$2x-3 = x-2 + 2\sqrt{x-2} + 1$$

$$2x-3-x+2-1 = 2\sqrt{x-2}$$

$$x-2 = 2\sqrt{x-2}$$

$$(x-2)^2 = (2\sqrt{x-2})^2$$

$$(x-2)(x-2) = 4(x-2)$$

$$x^2 - 2x - 2x + 4 = 4x - 8$$

$$x^2 - 4x + 4 - 4x + 8 = 0$$

$$x^2 - 8x + 12 = 0$$

$$(x-2)(x-6) = 0$$

$$\sqrt{x-2} \quad \sqrt{x-6}$$

$$\{2, 6\}$$

Introduction to Complex numbers:

Complex number $\Rightarrow a + bi$

$i = \sqrt{-1}$
 $i^2 = -1$

\uparrow \uparrow
 Real Imaginary
 Part Part

$3 - 2i$	}	$-\frac{2}{5} + \frac{1}{5}i$
Real Part = 3		Re. Part = $-\frac{2}{5}$
Imag. Part = -2		Im. Part = $\frac{1}{5}$

$-8i$
 Re. Part = 0
 Im. Part = -8

It is common to use
 Z for complex number
 $Z = a + bi$

If $Z = a + bi$, the conjugate of Z is
 $a - bi$.

$$\bar{Z} = a - bi$$

$$Z = 3 - 2i$$

$$\bar{Z} = 3 + 2i$$

$$Z = -5 + 7i$$

$$\bar{Z} = -5 - 7i$$

$$Z = -12i$$

$$\bar{Z} = 12i$$

$$Z = \frac{3}{5} - \frac{4}{5}i$$

$$\bar{Z} = \frac{3}{5} + \frac{4}{5}i$$

Operations with Complex numbers:

$$1) \sqrt{-25} = \sqrt{25} \sqrt{-1} = 5i = \boxed{5i}$$

$$2) -\sqrt{-18} = -\sqrt{9} \sqrt{2} \sqrt{-1} \\ = -3\sqrt{2} i = \boxed{-3\sqrt{2}i} = \boxed{-3i\sqrt{2}}$$

$$3) \sqrt{75} - 3\sqrt{-64} \\ = \sqrt{25} \sqrt{3} - 3\sqrt{64} \sqrt{-1} \\ = 5\sqrt{3} - 3 \cdot 8 i = \boxed{5\sqrt{3} - 24i}$$

$$4) 2(3 - 2i) + 5(1 + 4i) \\ = 6 - 4i + 5 + 20i \\ = \boxed{11 + 16i} \quad \begin{array}{l} \text{Re. Part} = 11 \\ \text{Im. Part} = 16 \end{array}$$

$$5) -3(4 - 2i) + 6(2 - 4i) \\ = -12 + 6i + 12 - 24i \\ = \boxed{-18i}$$

$$6) 4i(1 + 2i) - 3(5 - 6i) \\ = \boxed{4i} + 8i^2 - 15 + \boxed{18i} \\ = 22i + 8(-1) - 15 \\ = 22i - 8 - 15 = 22i - 23 \\ = \boxed{-23 + 22i}$$

$$7) (3 + 2i)(2 + 5i) \quad \text{Hint: FOIL \& Simplify}$$

$$= 6 + 15i + 4i + 10i^2$$

$$= 6 + 19i + 10(-1)$$

Re Part -4

Im. Part 19

$$= 6 + 19i - 10 = \boxed{-4 + 19i}$$

$$8) (3 - 4i)(3 + 4i)$$

$$= 9 - 12i - 12i - 16i^2 = 9 - 16(-1)$$

$$= 9 + 16 = \boxed{25}$$

$$9) (2 + 5i)^2$$

Hint: $(2+5i)^2 = (2+5i)(2+5i)$

$$= (2 + 5i)(2 + 5i)$$

$$= 4 + 10i + 10i + 25i^2$$

$$= 4 + 20i + 25(-1)$$

$$= 4 + 20i - 25 = \boxed{-21 + 20i}$$

$$10) (4 - 3i)^2 = (4 - 3i)(4 - 3i)$$

$$= 16 - 12i - 12i + 9i^2$$

$$= 16 - 24i + 9(-1)$$

$$= 16 - 24i - 9 = \boxed{7 - 24i}$$

Re Part = 7

Im Part = -24

Multiply top and bottom by the conjugate of the denominator, then simplify

$$\begin{aligned}
 11) \quad \frac{2}{1+i} &= \frac{2(1-i)}{(1+i)(1-i)} = \frac{2-2i}{1-\cancel{i}+\cancel{i}-i^2} \\
 &\text{Foil \& Simplify} \\
 &= \frac{2-2i}{1-(-1)} = \frac{2-2i}{1+1} = \frac{2-2i}{2} \\
 &= \frac{2}{2} - \frac{2}{2}i = \boxed{1-i}
 \end{aligned}$$

$$\begin{aligned}
 12) \quad \frac{10i}{3-4i} &= \frac{10i(3+4i)}{(3-4i)(3+4i)} \\
 &= \frac{30i+40i^2}{9+\cancel{12i}-\cancel{12i}-16i^2} \\
 &= \frac{30i+40(-1)}{9-16(-1)} \\
 &= \frac{30i-40}{9+16} = \frac{30i-40}{25} \\
 \text{Re. Part} &= -\frac{8}{5} \\
 \text{Im. Part} &= \frac{6}{5} \\
 &= \frac{30}{25}i - \frac{40}{25} = \frac{6}{5}i - \frac{8}{5} = \boxed{-\frac{8}{5} + \frac{6}{5}i} \checkmark
 \end{aligned}$$

$$13) \frac{2-3i}{2+3i} = \frac{(2-3i)(2-3i)}{(2+3i)(2-3i)}$$

$$= \frac{4 - 6i - 6i + 9i^2}{4 - \cancel{6i} + \cancel{6i} - 9i^2}$$

$$\text{Re. Part} = \frac{-5}{13} = \frac{4 - 12i + 9(-1)}{4 - 9(-1)} = \frac{4 - 12i - 9}{4 + 9}$$

$$\text{Im. Part} = \frac{-12}{13} = \frac{-5 - 12i}{13} = \boxed{\frac{-5}{13} - \frac{12i}{13}}$$

Now powers of i

1) Even Powers

$$i^{200} = (i^2)^{100} = (-1)^{100} = \boxed{1}$$

$$i^{150} = (i^2)^{75} = (-1)^{75} = \boxed{-1}$$

2) odd powers

$$i^{21} = i^{20} \cdot i = (i^2)^{10} \cdot i = (-1)^{10} \cdot i = 1 \cdot i = \boxed{i}$$

$$i^{35} = i^{34} \cdot i = (i^2)^{17} \cdot i = (-1)^{17} \cdot i = -1 \cdot i = \boxed{-i}$$

$$i^{123} = i^{122} \cdot i = (i^2)^{61} \cdot i = (-1)^{61} \cdot i = -1 \cdot i = \boxed{-i}$$